

PreCalculus  
Worksheet 4.1

1. The following angles are given to you in radian measure. Without converting to degrees, draw a sketch of each angle in standard position AND give the reference angle.

a)  $\frac{8\pi}{5}$

b)  $\frac{\pi}{6}$

c)  $\frac{13\pi}{9}$

d)  $\frac{3\pi}{2}$

e) 5.6

f) 1.3

2. If you are given a degree measurement, how do you convert it to radians?

3. If you are given a radian measurement, how do you convert it to degrees?

4. Convert the following angle measurements from degrees to radians (or vice-versa).

a)  $\frac{2\pi}{3}$

b)  $24^\circ$

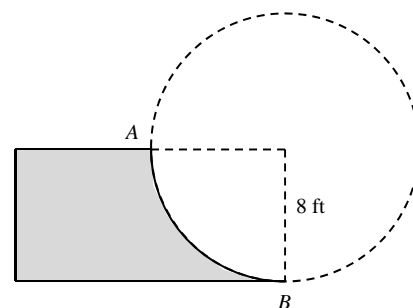
c) 4.6

d)  $310^\circ$

5. What is the formula for finding arc length? What does each letter represent?

6. A sector has an arc length 12 cm and a central angle of  $\frac{\pi}{3}$  radians. Find the radius of the circle.

7. The skateboard ramp at the right is called a quarter pipe. The curved surface is determined by the radius of a circle. Find the length of the curved part of the ramp.



8. The paddlewheel of a riverboat has a diameter of 24 feet. Find the arc length of the circle made when the paddlewheel rotates  $300^\circ$ .

9. How many degrees are in one revolution? How many degrees in  $\frac{1}{8}$  of a revolution?

10. How many radians are in one revolution? How many radians in  $\frac{3}{4}$  of a revolution?

11. The radius of a car wheel is 13 inches. How many revolutions per minute is the wheel making when the car is traveling at 40 mph.

12. A speedometer on a car is calibrated for the specific tire size for which the car was built. The speedometer uses the revolutions per minute of the tires to calculate the speed. If the same car in the last question is “tricked out” to have 15 inch wheels and its wheels are turning the same revolutions per minute as you found above, now how fast is the car actually traveling in mph?

13. Monster Truck tires have a radius of 33 inches. How far does a monster truck travel in feet after just three-fourths of a tire rotation?

14. A radial arm saw has a circular cutting blade with a diameter of 10 inches. It spins at 2000 rpm (revolutions per minute). If there are 12 cutting teeth per inch on the cutting blade, how many teeth cross the cutting surface each second?

There are two other ways to measure angles not discussed in the notes. The first is Degrees, Minutes, Seconds (DMS). Map coordinates are given using latitude and longitude in terms of degrees, minutes, seconds or in terms of a decimal form of the degree. We use the minutes and seconds notation to replace the decimal points on a degree.

We denote 53 degrees, 20 minutes, and 19 seconds as  $53^\circ 20'19''$ .

You can use your TI-83 calculator to convert from DMS to degrees and vice-versa.

To convert from DMS to degrees, simply type in the symbols for degrees (2<sup>nd</sup> APPS, 1), minutes (2<sup>nd</sup> APPS, 2), and seconds (ALPHA, +) and pressing ENTER.

15. Convert  $53^\circ 20'19''$  to degrees to the nearest thousandth.

To convert from degrees to DMS, simply enter the degree in your calculator followed by ►DMS (2<sup>nd</sup> APPS, 4)

16. Convert  $34.843^\circ$  to DMS.

Another way to measure angles is bearings. In navigation, the course or *bearing* of an object is sometimes given as the angle of the line of travel measured clockwise from due north. In other words, the initial side of a bearing is north.

17. Find the angle in degrees that describes the compass bearing.

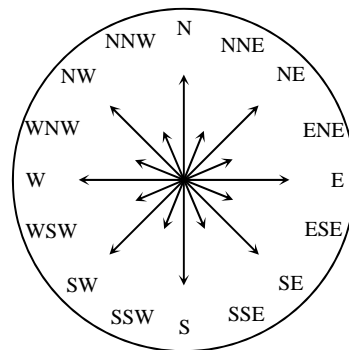
a) Bearing "SE"

b) Bearing "WNW"

18. Find the compass bearing closest to each angle.

a) Bearing  $145^\circ$

b) Bearing  $318^\circ$

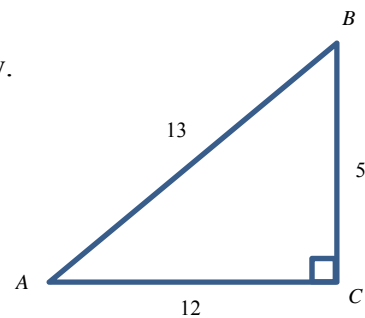


19. The captain of the tourist boat *Julia* out of Oak Harbor follows a bearing of  $38^\circ$  for 2 miles and then changes to a bearing of  $47^\circ$  course for the next 4 miles. Draw a sketch of this trip.

20. Convert a bearing of  $140^\circ$  into a radian measurement.

PreCalculus  
Worksheet 4.2

1. Find the six trigonometric ratios of angle  $A$  in the triangle below.



**In questions 2 and 3, assume that the given angle is an acute angle in a right triangle satisfying the given conditions. Evaluate the other 5 trigonometric functions.**

2.  $\cot \theta = \frac{11}{3}$

3.  $\csc \theta = \frac{12}{5}$

**For questions 4 – 9, evaluate each *without using a calculator*.**

4.  $\sin\left(\frac{\pi}{3}\right)$

5.  $\tan\left(\frac{\pi}{4}\right)$

6.  $\cot\left(\frac{\pi}{6}\right)$

7.  $\sec\left(\frac{\pi}{3}\right)$

8.  $\cos\left(\frac{\pi}{4}\right)$

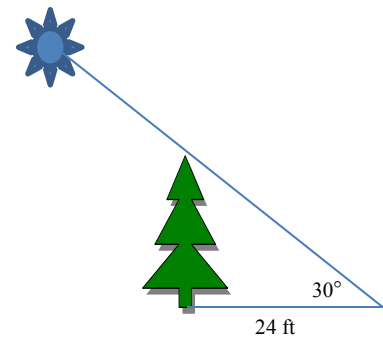
9.  $\csc\left(\frac{\pi}{3}\right)$

10. If  $\sin \theta = \frac{\text{opposite}}{\text{hypotenuse}}$  and  $\cos \theta = \frac{\text{adjacent}}{\text{hypotenuse}}$ , what is  $\frac{\sin \theta}{\cos \theta}$  ?

11. Complete the following table using the two right triangles and the result from the last question.  
 ...YOU NEED TO MEMORIZE THESE VALUES BEFORE NEXT CLASS!  
 (more on this at the optional question at the end)

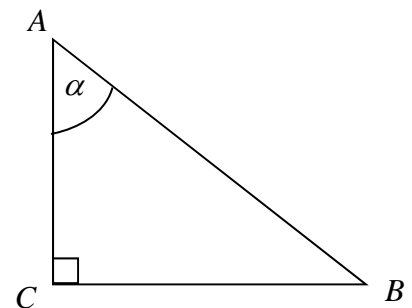
$\theta \rightarrow$	$30^\circ$	$45^\circ$	$60^\circ$
$\sin \theta$			
$\cos \theta$			
$\tan \theta$			

12. [No Calculator] A tree casts a shadow approximately 24 feet long when the sun's rays form a  $30^\circ$  angles with the earth. How tall is the tree?

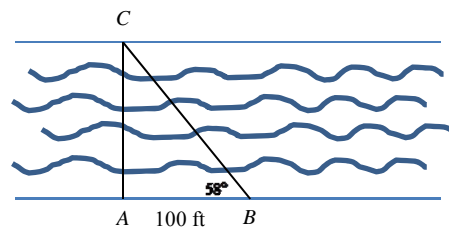


13. [No Calculator] A rope tied to the top of a tent pole on one end and nailed to the ground for support on the other end is 10 feet long and makes a  $45^\circ$  angle with the ground. How tall is the tent pole?

14. [Calculator Required] Solve  $\triangle ABC$  for all of its unknown parts when  $a = 10.3$  and  $\alpha$  (alpha)  $= 25^\circ$ .



15. [Calculator Required] A biologist wants to know the width of a river in order to properly set instruments for studying the pollutants in the water. From point A, the biologist walks downstream 100 feet and sights to point C. From this sighting, it is determined that  $m\angle ABC = 58^\circ$ . How wide is the river?



16. Find the acute angle  $\theta$  that satisfies each given equation. Give  $\theta$  in both radians and degrees. You should be able to do this WITHOUT the use of a calculator!

a)  $\sin \theta = \frac{1}{2}$

b)  $\sin \theta = \frac{\sqrt{3}}{2}$

c)  $\cot \theta = \frac{1}{\sqrt{3}}$

d)  $\cos \theta = \frac{\sqrt{2}}{2}$

e)  $\sec \theta = 2$

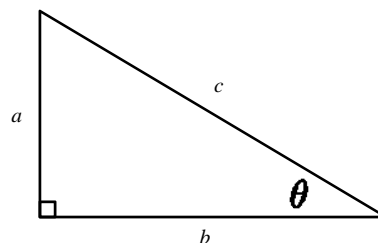
f)  $\cot \theta = 1$

g)  $\tan \theta = \frac{\sqrt{3}}{3}$

h)  $\cos \theta = \frac{\sqrt{3}}{2}$

h)  $\tan \theta = 1$

17. Using the triangle below, PROVE that if  $\theta$  is an acute angle in any right triangle, then  $(\sin \theta)^2 + (\cos \theta)^2 = 1$ .



...Optional ...

The chart below can be “memorized” without the triangles if you learn a couple of patterns ...

Step 1: Make sure the chart is setup like it is below  
... (30/45/60) and (sine,cosine,tangent)

Step 2: Enter 1, 2, and 3, in the first row ... and  
3, 2, 1 in the second row

Step 3: Take the square root of all the numbers from step 2 ...  
keep in mind that  $\sqrt{1} = 1$

Step 4: Divide ALL 6 of the numbers in the first two rows by 2.

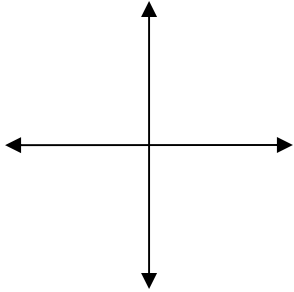
$\theta \rightarrow$	$30^\circ$	$45^\circ$	$60^\circ$
$\sin \theta$			
$\cos \theta$			
$\tan \theta$			

Step 5: Use the fact that  $\tan \theta = \frac{\sin \theta}{\cos \theta}$  to generate the values in the last row.

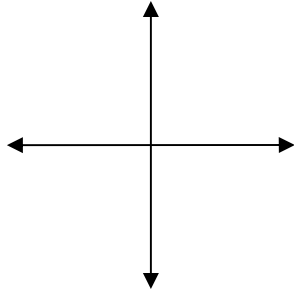
Pre Calculus  
Worksheet 4.3 day 1

1. Graph each angle in standard position and give the reference angle. Always give  $\theta_{ref}$  in the same units as the given angle

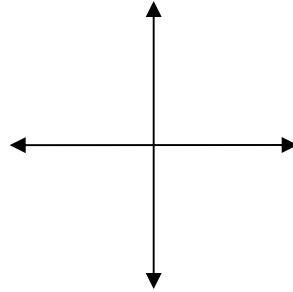
a)  $-135^\circ$



b)  $\frac{11\pi}{3}$



c)  $-2$



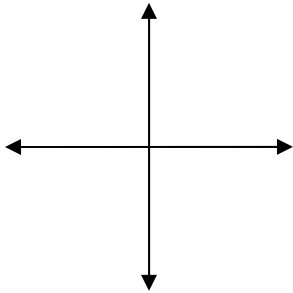
2. Find one angle that is NOT co-terminal with the others listed.

a)  $150^\circ, 510^\circ, -210^\circ, 450^\circ, 870^\circ$

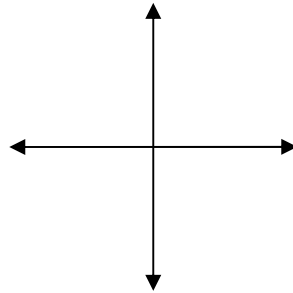
b)  $\frac{5\pi}{3}, -\frac{5\pi}{3}, \frac{11\pi}{3}, -\frac{7\pi}{3}, \frac{365\pi}{3}$

3. Find and draw one positive **and** one negative angle that is co-terminal with the given angle.

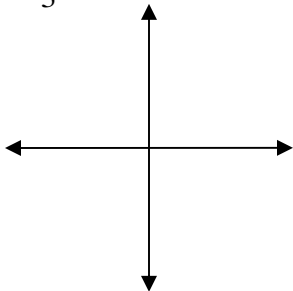
a)  $30^\circ$



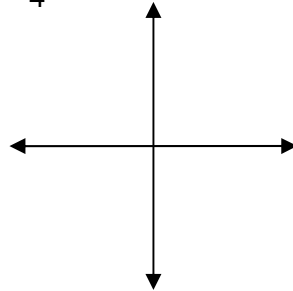
c)  $-420^\circ$



d)  $\frac{4\pi}{3}$



f)  $-\frac{23\pi}{4}$





4. Point  $P$  is on the terminal side of angle  $\theta$ . Evaluate the six trigonometric functions for  $\theta$ . If a function is undefined, write undefined.

a)  $P(-5, 12)$

b)  $P(6, -6)$

5. Remembering your 45–45–90 right triangle and the sine and cosine of  $45^\circ$ , explain how you could have predicted the answers for question 4b.

6. Determine the sign (+ or –) of each expression WITHOUT using a calculator.

a)  $\cos 143^\circ$

b)  $\sin \frac{7\pi}{8}$

c)  $\tan 192^\circ$

d)  $\sec\left(-\frac{4\pi}{5}\right)$

7. Find an interval of values on the domain  $[0, 2\pi]$  for  $\theta$  that makes the following sets of statements true.

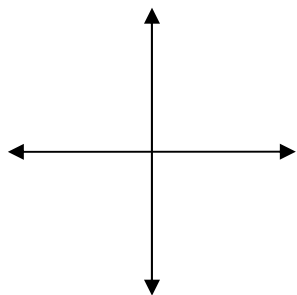
a)  $\csc \theta > 0$  and  $\cos \theta < 0$

b)  $\tan \theta < 0$  and  $\sec \theta > 0$

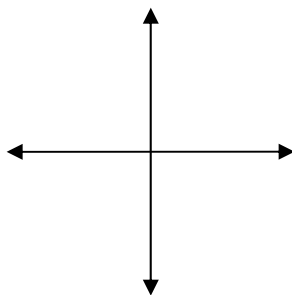
The next group of questions are designed to help you make some connections about some special theorems from trigonometry *WITHOUT* actually going through a proof of the theorems...you're welcome. ☺ They are easiest to approach from a graphical perspective, so we will start there. Finishing this page by next class will earn you some extra credit!

8. Graph any acute radian angle  $\theta$ . Then, use that picture to find where each requested angle would be located. Remember, you know how big  $\pi$  radians is in terms of a revolution.

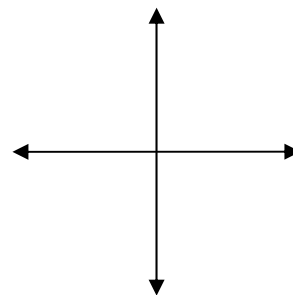
a) Graph  $(-\theta)$



b) Graph  $(\pi + \theta)$



c) Graph  $(\pi - \theta)$



9. Using the graphs above, draw reference triangles to the  $x$  axis. Label the sides of your triangles given that  $\sin \theta = 0.12$  and  $\cos \theta = 0.05$  for the original angle  $\theta$ . Next, find the value of each expression. Remember All Students Take Classes!!

a)  $\sin(-\theta) =$

b)  $\sin(\pi + \theta) =$

c)  $\sin(\pi - \theta) =$

$\cos(-\theta) =$

$\cos(\pi + \theta) =$

$\cos(\pi - \theta) =$

Let's see if you "get it"...

10. If  $\cos \theta = -0.4$ , find  $\cos(\pi + \theta)$ .

11. If  $\sin \theta = 0.4$ , find  $\sin(-\theta) + \csc \theta$  without a calculator.

One last idea that will be very important next chapter...

12. We know the radius of the Unit Circle is 1 and angle  $\theta$  makes a reference triangle with a right angle. We further know that any missing side of a right triangle can be found with the Pythagorean Theorem and that the sides of the right triangle are  $x$  and  $y$ . If you know that  $\sin \theta = y$  and  $\cos \theta = x$ , what is the relationship between  $\sin \theta$  and  $\cos \theta$ ?

13. Find  $\sin \theta$  when  $\cos \theta = -0.2$ .

Pre Calculus  
Worksheet 4.3 day 2

NO CALCULATOR

*Graph each angle and list  $\theta_{ref}$ . Then, find the exact value of each trigonometric function.*

1.  $\sin\left(\frac{\pi}{6}\right)$

2.  $\sin\left(-\frac{5\pi}{6}\right)$

3.  $\sin\left(\frac{11\pi}{6}\right)$

4.  $\sin\left(\frac{5\pi}{6}\right)$

5.  $\sin\left(\frac{13\pi}{6}\right)$

6.  $\sin\left(\frac{7\pi}{6}\right)$

7. What do you notice about the reference angles for questions 1-6? What do you notice about the answers?

8. Do you think a similar pattern would occur if questions 1-6 asked for cosines? For tangents?

9. What is a quadrantal angle? Explain why the 30-60-90 or 45-45-90 right triangles are NOT helpful on quadrantal angles.

Find the exact value of each trigonometric function.

10.  $\cos\left(\frac{2\pi}{3}\right)$

11.  $\tan(3\pi)$

12.  $\csc\left(\frac{11\pi}{6}\right)$

13.  $\sec 135^\circ$

14.  $\sin(-270^\circ)$

15.  $\cot(225^\circ)$

16.  $\sin\left(\frac{7\pi}{4}\right)$

17.  $\tan\left(\frac{-5\pi}{6}\right)$

18.  $\sec(-45^\circ)$

19.  $\sec(-90^\circ)$

20.  $\tan(270^\circ)$

21.  $\csc(\pi)$

22. What do you notice about your answers to questions 19-21? Why does this happen??

23. If  $\cos \theta = -\frac{5}{13}$  and  $\tan \theta > 0$ , find  $\sin \theta$ .#

24. If  $\cot \theta = -\frac{15}{8}$  and  $\cos \theta < 0$ , find  $\csc \theta$ .

25. If  $\sec \theta = \frac{5}{2}$  and  $\sin \theta < 0$ , find  $\cot \theta$ .